# Geometric Algebra
## New Foundations, New Insights

SIGGRAPH 2001 Course

Organizers: Ambjorn Naeve and Alyn Rockwood

## Description

Geometric Algebra has developed in the last decades from earlier pioneering mathematics of Grassmann and Clifford. It promises to stimulate new methods and insights in all areas of science and engineering dealing with spatial relationships, including computer graphics and related fields. This course first introduces and motivates the topic, and then provides example applications of interest for computer graphics professionals and researchers.

Geometric Algebra unifies many different and redundant mathematical systems in current use. It is especially useful for handling geometric problems, since it allows for intrinsic, i.e., coordinate free, and dimensionally seamless descriptions of geometry. It has wide application in computer graphics, e.g., kinematics and dynamics, simplicial calculations (polygons, FEM), fluid flow, collision detection, quaternion splines , elastic deformations, curve and surface definition, vector fields etc. In all cases, new insights and improved algorithms invariably result. Geometric Algebra is a new and fundamental language for the mathematics of computer graphics, as well as for modeling and interactive techniques in general.

## Speakers

Chris Doran, Dept. of Physics, Cambridge University, UK
Leo Dorst, Dept. of Computer Science, University of Amsterdam, the Netherlands
David Hestenes, Dept. of Physics, Arizona State University, Arizona
Joan Lasenby, Dept. of Engineering, Cambridge University, UK
Stephen Mann, Dept. of Computer Science, Waterloo University, Ontario
Ambjorn Naeve, Computing Science, Royal Institute of Technology, Sweden
Alyn Rockwood, Mitsubishi Electric Research Lab, Boston MA.

# Table of Contents / Schedule

## Biographies

**Chris Doran** studied at Cambridge University, obtaining a Distinction in Part III Mathematics and a Ph.D. in 1994. He was elected a Junior Research Fellow of Churchill College in 1993, and was made a Lloyd's of London Fellow in 1996. He currently holds an EPSRC Advanced Fellowship, and is a Fellow of Sidney Sussex College, Cambridge. Chris has published widely on aspects of mathematical physics and is currently researching the applications of geometric methods in Engineering. His interests include geometric algebra, computer vision, robotics, general relativity, and quantum field theory.

**Leo Dorst** received a Ph.D. 1986 from the Applied Physics Department, Delft University of Technology, The Netherlands. His thesis was on accurate geometrical measurements in discretized images. From 1986 to 1992 he worked as senior research scientist at Philips Laboratories, Briarcliff Manor, NY, USA, focusing on robot path planning and task abstraction in goal-directed systems. From 1992 he has been an assistant professor at the University of Amsterdam, The Netherlands, where his continued interest is on planning and representation in autonomous systems, with an emphasis on reasoning with uncertainty, and the use of Geometric Algebra for geometric representation and computation.

**David Hestenes** is professor of physics, Arizona State University. His principal work has been the programmatic development of Geometric Algebra as a unified mathematical language for science and engineering. Researchers in leading institutions throughout the world are now applying GA to improve software in commercial and government space programs, to biomechanics/robotics and computer vision, to scientific visualization, to conformations of complex molecules, to relativistic quantum theory and to general relativity (3 books & more than 40 papers). He is a fellow of the American Physical Society and an Overseas Fellow of Churchill College (Cambridge) and a *Foundations of Physics* Honoree.

**Joan Lasenby** studied mathematics at Cambridge University graduating with first class honours in 1981. She gained a Distinction in Part III mathematics in 1983 and a PhD in Radio Astronomy in 1987. She held a Junior Research Fellowship at Trinity Hall College from 1986-9 and worked for Marconi Research Laboratory from 1989-90. She is currently a Royal Society University Research Fellow in the Signal Processing Group of the Cambridge University Engineering Department and a Fellow of Trinity College. Her research interests include applications of geometric algebra in computer vision and robotics, motion analysis and capture, constrained optimization and structural mechanics.

**Stephen Mann** is an associate professor of computer science at the University of Waterloo where he teaches computer graphics and splines. He received a Ph.D. and M. S. from the University of Washington and a B.A. from the University of California, Berkeley. He has recently spent half a year at the University of Amsterdam working with Geometric Algebra, where he co-developed GABLE. Current interests include splines, surface pasting, blossoming, triangular interpolants, and geometric algebra.

**Ambjorn Naeve** received a Ph.D. in Computational Geometry, 1993 from the Royal Institute of Technology (KTH), Stockholm. He has been an early advocate of using projective geometry in computational vision. Since 1984 he works with the Computer Vision and Active Perception research group at KTH, where he has headed the development of a number of computer programs that enhance geometrical understanding in various ways - including PDB. Since 1996 Ambjorn also works as a researcher at the Centre for user-oriented Information technology Design (CID) at KTH, where he is concerned with developing principles for the design of interactive learning environments.

**Alyn Rockwood** received his Ph.D. in applied mathematics from Cambridge University. He has spent 25 years in industrial and academic research, at SGI he developed the NURBS rendering methods for GL/OpenGL; at Evans and Sutherland he developed the first hardware textured graphics system; at Shape Data Ltd. he developed the first commercial automatic blending methods in CAD/CAM and more recently at Arizona State University he was a faculty member and project co-director for a major research project in brain imaging. He has authored several books and 50 articles on computer graphics, and served as the 1999 SIGGRAPH papers' chair.

## Relevant web-sites

that provide hands-on experience and extended resources:

1. http://www.cgl.uwaterloo.ca/~smann/GABLE/
   The hands-on interactive tutorial that will be demonstrated in the course. It requires a copy of MATLAB. It will also be available at the CAL.

2. http://modelingnts.la.asu.edu/GC_R&D.html
   Hestenes' web-site with many of the latest research papers in a wide variety of areas and links to other sites.

3. http://www.mrao.cam.ac.uk/~clifford/
   This is the Cambridge University site.

4. http://www-sigproc.eng.cam.ac.uk/vision
   Mainly computer vision applications (GA and non-GA).

5. http://cid.nada.kth.se/il
   Several interactive mathematical learning environments (including GA).